Two-Phase Numerical Simulation of Sediment Suspension Problems and its Application to Environmental Flow Modeling

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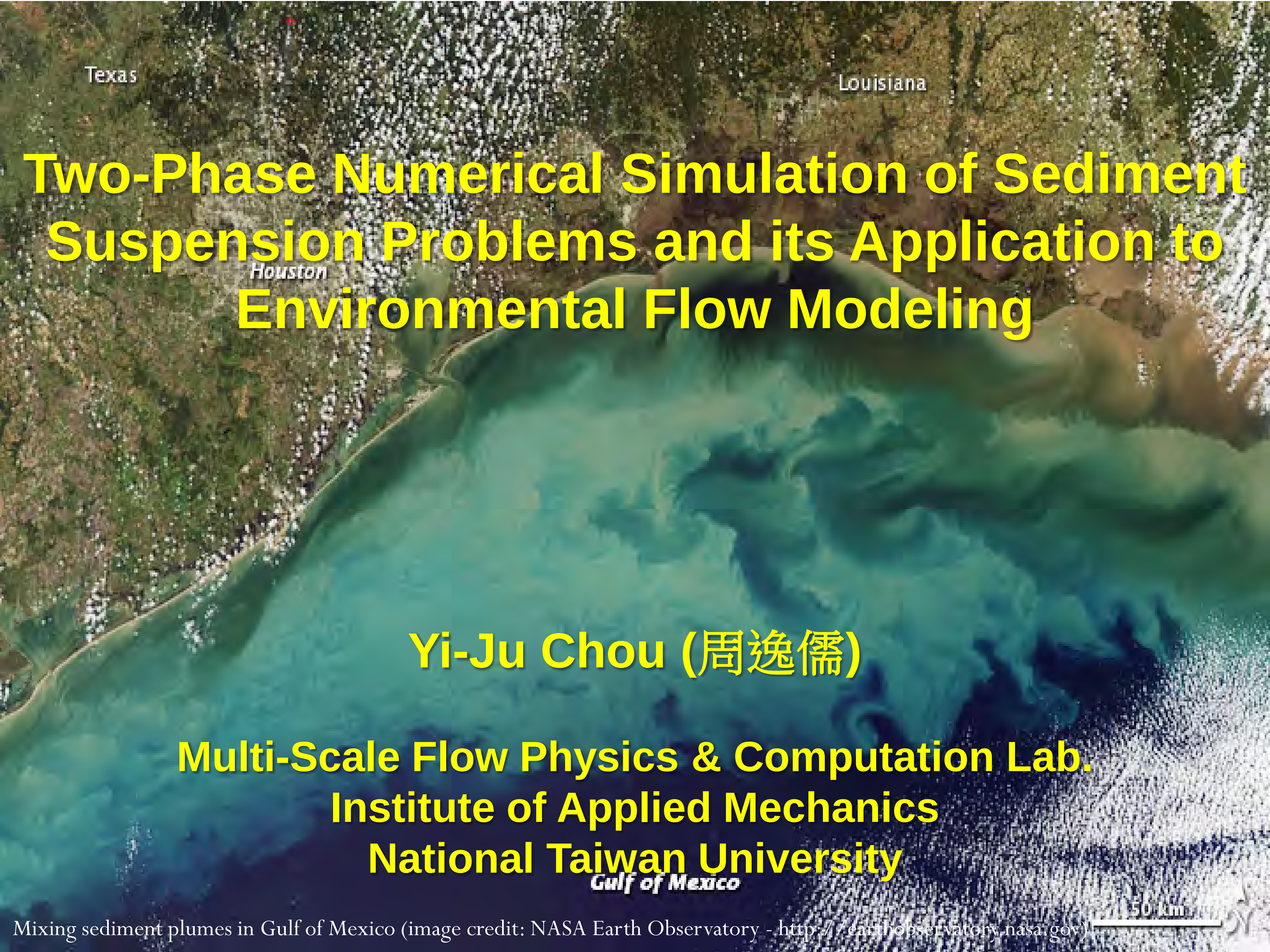
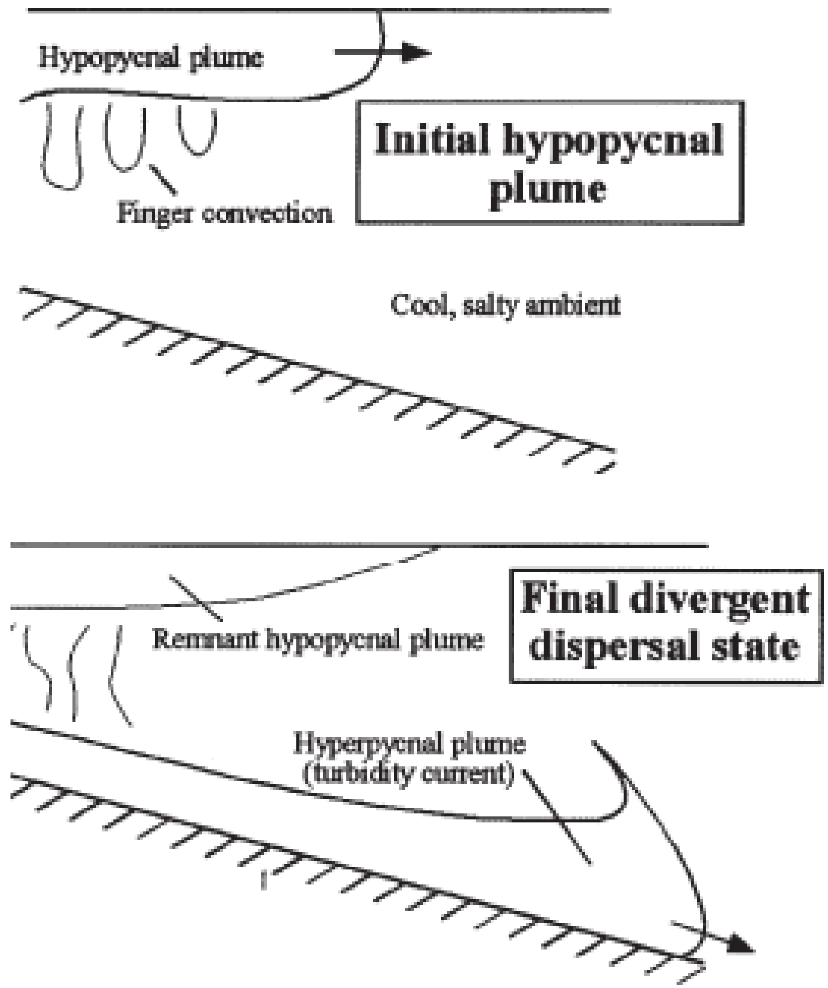


Mixing sediment plumes in Gulf of Mexico (image credit: NASA Earth Observatory - http://earthobservatory.nasa.gov)

Sediment River Plume

- Ocean acidification
- Phytoplankton bloom
- Subaqueous ecology

image credit: NASA Earth Observatory - http://earthobservatory.nasa.gov)

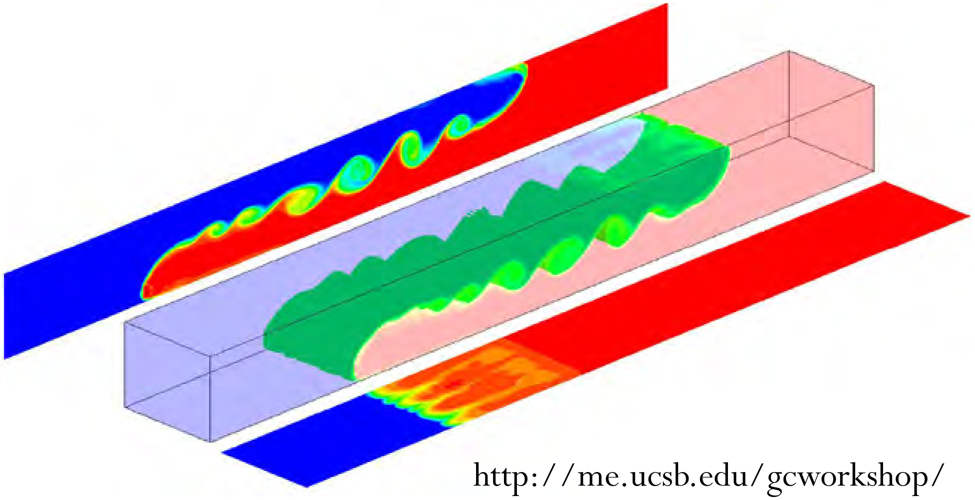


Parsons et al., Sedimentology, 2001

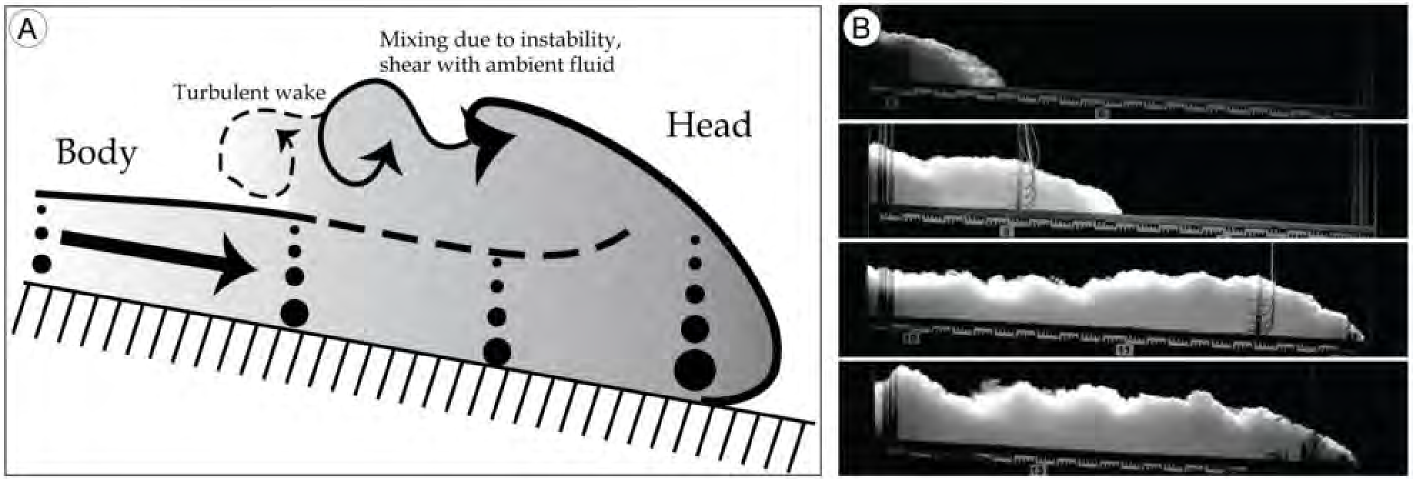
Turbidity Current

Implications

- Structure safety (e.g. pipelines)
- Coastal morphology
- Costal hazard



http://me.ucsb.edu/gcworkshop/

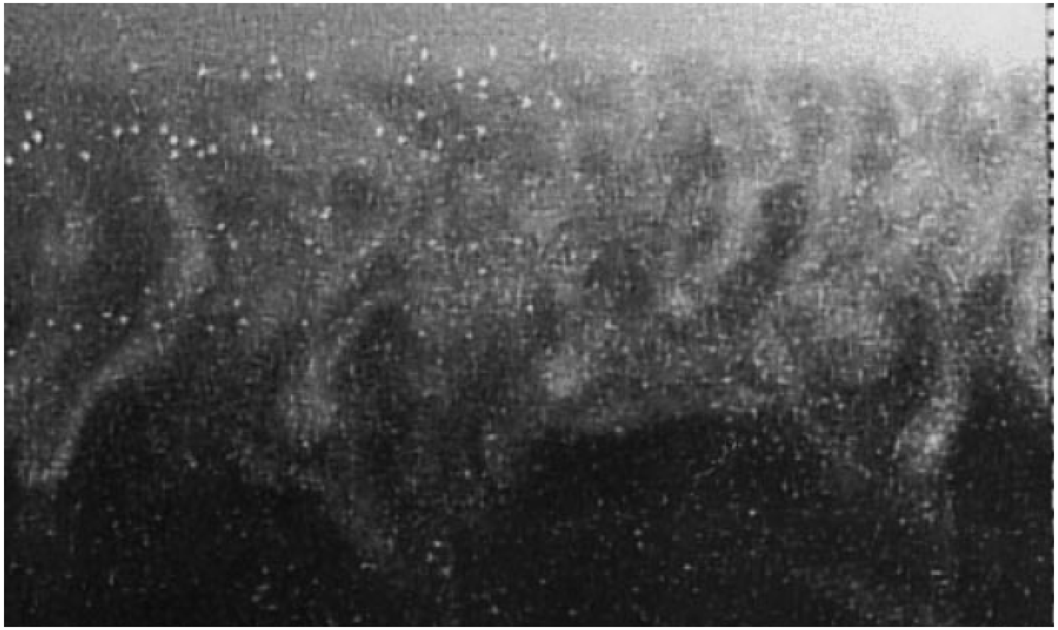


http://offtheshelfedge.wordpress.com/2012/11/07/the-bouma-sequence-and-turbidite-deposits/

Convective Instability

Implications

- Subaqueous ecology
- Water quality



Parsons et al., Sedimentology, 2001

Outline

- Physics background
- Review of the current method
- Objectives
- Numerical examples of RT instability
- Summary

Physics Background

Important characteristics

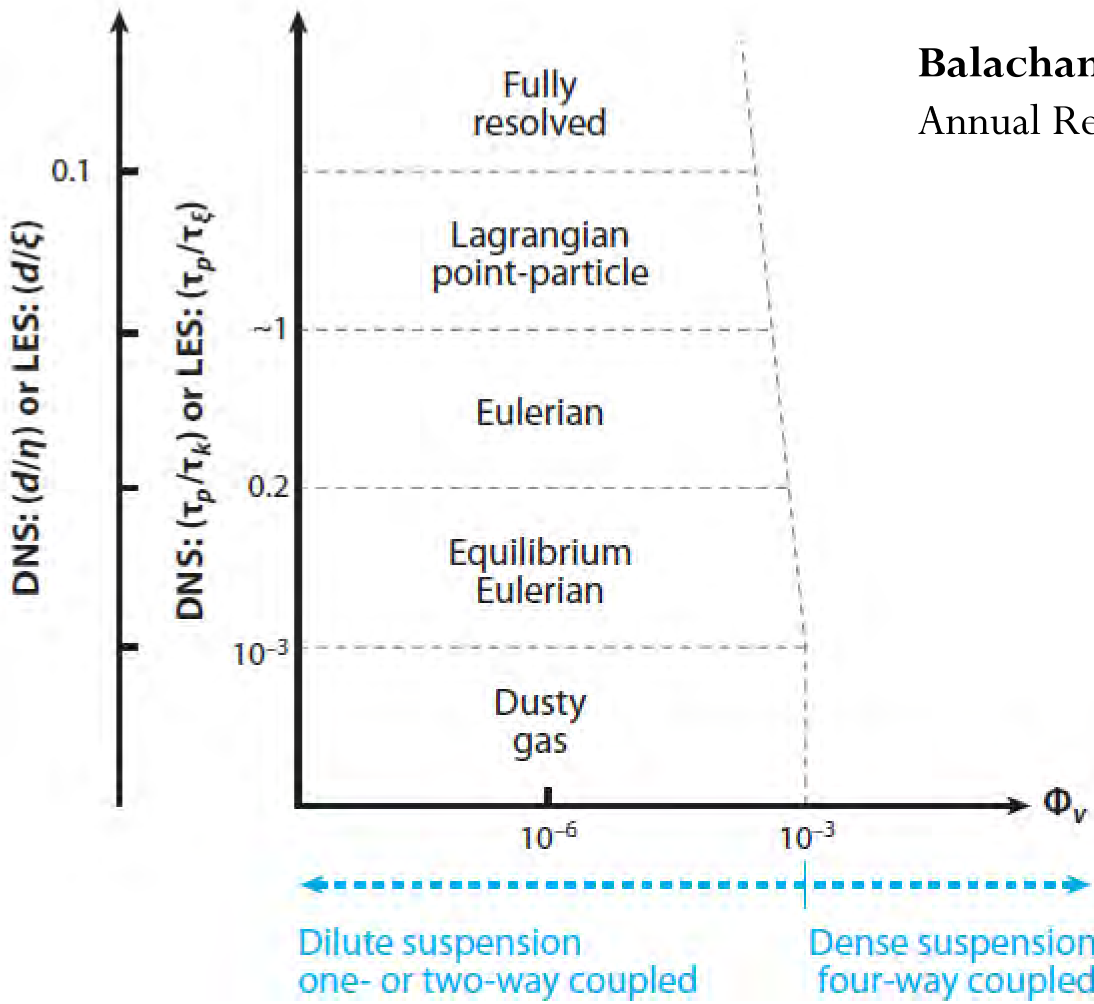
- Particle diameter ~ O(1-100 μm)
- Density ratio~ O(1)
- Turbulent flow
- Concentration (volume fraction) ~ O(0.01)
- Scale of interest ~ O(cm-km)

Important parameters

- Particle Reynolds number: $Re_p = \frac{|\mathbf{u}_c - \mathbf{u}_d| d_0}{\nu} < O(100)$ ➡ ***Stokes drag***
- Particle relaxation(response) time: $\tau_p = \frac{\rho_d}{\rho_c}\frac{{d_0}^2}{18\nu} \sim 10^{-5} - 10^{-3}$ sec
- Stokes number: $\frac{\tau_p}{\tau_k} < O(1)$ (τ_k: Kolmogorov time scale)
- Bagnold number: $Ba = \frac{\rho_s d_s \lambda^{1/2} \gamma}{\mu} \sim O(1) << 40$ ➡ ***Collision negligible***

Physics Background

Modeling strategy for dispersed two-phase turbulent flows :



Balachandar & Eaton,
Annual Rev. Fluid Mech., 2010

Physics Background

Motion of a sediment grain :

$$\frac{d\boldsymbol{u}_p}{dt} = \underbrace{\frac{\boldsymbol{u}_{c}-\boldsymbol{u}_p}{\tau_p}}_{\text{Drag}} \underbrace{- \frac{1}{s}\nabla p}_{\text{Pressure}} + \underbrace{\frac{C_{vm}}{s}\left(\frac{D\boldsymbol{u}_c}{Dt} - \frac{d\boldsymbol{u}_p}{dt}\right)}_{\text{Added mass}} + \underbrace{\left(1 - \frac{1}{s}\right)\boldsymbol{g}}_{\text{Gravity}}$$

+ Basset history term

+ Saffman lift force

$s = \frac{\rho_p}{\rho_c}$: Density ratio

C_{vm} : Added-mass coefficient (1/2 for the sphere)

$\tau_p = \frac{4}{3}\frac{s d_p}{C_D\left|\boldsymbol{u}_c - \boldsymbol{u}_p\right|}$: Particle relaxation time

Review of Existing Method

- *Single-phase method:*

$$\frac{\partial \overline{\mathbf{u}}_c}{\partial t} + \overline{\mathbf{u}}_c \cdot \nabla \overline{\mathbf{u}}_c = -\nabla \overline{p} + \boxed{\phi s \mathbf{g}'} + \nu \nabla^2 \overline{\mathbf{u}}_c$$

$$\overline{\mathbf{u}}_d = \overline{\mathbf{u}}_c - \boxed{w_{s,0} \hat{\mathbf{e}}_3}$$

Equilibrium state:
A balance between drag and gravitational force

$$\boxed{\nabla \cdot \overline{\mathbf{u}}_c = 0}$$

Scalar limit:
Zero-volume for particles

$$\boxed{\frac{\partial \phi}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}_c \phi) - \frac{\partial}{\partial z}(w_s \phi) = 0}$$

Has been employed to study a number of problems related to fine suspensions using DNS, LES, RANS

Objectives

What are we trying to answer?

- How can the equilibrium state be a good approximation?
- In what conditions are these conditions valid, or violated?
- What else are we missing, and how do they affect bulk mixing?
- Can we improve the current model without extra computational effort?

Method

Mathematical formulation *(Chou et al., 2014a)*

$$\frac{\partial}{\partial t}\begin{bmatrix}\overline{\mathbf{u}}_d\\ \overline{\mathbf{u}}_c\end{bmatrix}+\begin{bmatrix}\overline{\mathbf{u}}_d\cdot\nabla\overline{\mathbf{u}}_d\\ \overline{\mathbf{u}}_c\cdot\nabla\overline{\mathbf{u}}_c\end{bmatrix}=\mathcal{A}\begin{bmatrix}\frac{\overline{\mathbf{u}}_c-\overline{\mathbf{u}}_d}{\tau_p}\\ -\frac{\phi}{1-\phi}s\frac{\overline{\mathbf{u}}_c-\overline{\mathbf{u}}_d}{\tau_p}\end{bmatrix}+\mathcal{A}\begin{bmatrix}\frac{s-1}{s}\mathbf{g}\\ 0\end{bmatrix}+\mathcal{A}\begin{bmatrix}-\frac{\nabla\overline{p}}{s}\\ -\nabla\overline{p}\end{bmatrix}$$

$$+\begin{bmatrix}\frac{\phi}{1-\phi}\mathcal{A}_{(1,2)}\nu\nabla^2\overline{\mathbf{u}}_d\\ \mathcal{A}_{(2,2)}\nu\nabla^2\overline{\mathbf{u}}_c\end{bmatrix}+\begin{bmatrix}\mathcal{A}_{(1,2)}\nu\nabla^2\overline{\mathbf{u}}_c\\ \frac{\phi}{1-\phi}\mathcal{A}_{(2,2)}\nu\nabla^2\overline{\mathbf{u}}_d\end{bmatrix}$$

$$+\begin{bmatrix}\mathcal{A}_{(1,2)}\frac{\nabla\phi}{1-\phi}\cdot\nu\nabla(\overline{\mathbf{u}}_d-\overline{\mathbf{u}}_c)-\mathcal{A}_{(1,2)}\frac{\nabla^2\phi}{1-\phi}\nu(\overline{\mathbf{u}}_d-\overline{\mathbf{u}}_c)\\ \mathcal{A}_{(2,2)}\frac{\nabla\phi}{1-\phi}\cdot\nu\nabla(\overline{\mathbf{u}}_d-\overline{\mathbf{u}}_c)-\mathcal{A}_{(2,2)}\frac{\nabla^2\phi}{1-\phi}\nu(\overline{\mathbf{u}}_d-\overline{\mathbf{u}}_c)\end{bmatrix}$$

$$-\begin{bmatrix}\nabla\cdot\overline{\Sigma}_{Re,d}\\ \nabla\cdot\overline{\Sigma}_{Re,c}\end{bmatrix}-\mathcal{A}\begin{bmatrix}\frac{\nabla\phi}{\phi}\cdot\overline{\Sigma}_{Re,d}\\ \frac{\nabla(1-\phi)}{1-\phi}\cdot\overline{\Sigma}_{Re,c}\end{bmatrix},$$

$$\mathcal{A}=\begin{bmatrix}1+\frac{C_{vm}}{s} & -\frac{C_{vm}}{s}\\ -C_{vm}\frac{\phi}{1-\phi} & 1+C_{vm}\frac{\phi}{1-\phi}\end{bmatrix}^{-1}=\frac{1}{1+\frac{C_{vm}}{s}+C_{vm}\frac{\phi}{1-\phi}}\begin{bmatrix}1+C_{vm}\frac{\phi}{1-\phi} & \frac{C_{vm}}{s}\\ C_{vm}\frac{\phi}{1-\phi} & 1+\frac{C_{vm}}{s}\end{bmatrix}$$

***s*: density ratio**

Method

Numerical solution procedure *(Chou et al., 2014a)*

- Predictor

  Solve momentum of two phases without pressure.

- Pressure Poisson solver

$$\frac{1}{\Delta t}\nabla\cdot\left\{\left[\phi^{n+1}\ \ 1-\phi^{n+1}\right]\begin{bmatrix}\overline{\mathbf{u}}_d^*\\ \overline{\mathbf{u}}_c^*\end{bmatrix}\right\}=\nabla\cdot\left\{\left[\phi^{n+1}\ \ 1-\phi^{n+1}\right]\mathcal{A}^{n+1}\begin{bmatrix}-\frac{1}{s}\nabla\overline{p}^{n+1}\\ -\nabla\overline{p}^{n+1}\end{bmatrix}\right\}$$

- Corrector

$$\begin{bmatrix}\overline{\mathbf{u}}_d^{n+1}\\ \overline{\mathbf{u}}_c^{n+1}\end{bmatrix}=\begin{bmatrix}\overline{\mathbf{u}}_d^*\\ \overline{\mathbf{u}}_c^*\end{bmatrix}+\Delta t\,\mathcal{A}^{n+1}\begin{bmatrix}-\frac{1}{s}\nabla\overline{p}^{n+1}\\ -\nabla\overline{p}^{n+1}\end{bmatrix}$$

Hindered settling

Theoretical result of steady-state gravitational settling of uniform concentration

$$\mathbf{0} = \mathcal{A}\begin{bmatrix} \frac{\bar{\mathbf{u}}_c - \bar{\mathbf{u}}_d}{\tau_p} + \mathbf{g}' \\ -\frac{\phi}{1-\phi} s \frac{\bar{\mathbf{u}}_c - \bar{\mathbf{u}}_d}{\tau_p} \end{bmatrix} + \mathcal{A}\begin{bmatrix} -\frac{\nabla \bar{p}}{s} \\ -\nabla \bar{p} \end{bmatrix}$$

$$\mathbf{g}' = \mathbf{g}(s-1)/s.$$

Assuming the difference is only in the vertical direction

$$\bar{\mathbf{u}}_d = \bar{\mathbf{u}}_c + w_s \hat{e}_3$$

Applying mixture incompressibility

$$\phi \mathbf{u}_d + (1-\phi)\mathbf{u}_c = \mathrm{const}$$

The hindered settling velocity is obtained

$$\mathbf{u}_d = (1-\phi)^2 \mathbf{g}' \tau_p = (1-\phi)^2 w_{s,0}$$

Deviation from the single-phase model

What are we missing? *(Chou et al., 2014b)*

- **Non-equilibrium particle inertia (NEPI)**

$$\overline{\mathbf{u}}_d = \overline{\mathbf{u}}_c - w_{s,0}\hat{\mathbf{e}}_3 + \tau_p \left(1 - \frac{1}{s}\right) \frac{D\overline{\mathbf{u}}_c}{Dt}$$

- **NEPI effect in the carrier flow (continuous phase)**

$$\frac{D\overline{\mathbf{u}}_c}{Dt} = \left.\frac{D\overline{\mathbf{u}}_c}{Dt}\right|_{eq} - \frac{\phi}{1-\phi}\tau_p \left(s - 1\right) \frac{D\overline{\mathbf{u}}_c}{Dt}$$

- **Mixture incompressibility**

$$\nabla \cdot \left[\phi \overline{\mathbf{u}}_d + (1-\phi)\,\overline{\mathbf{u}}_c\right] = 0$$

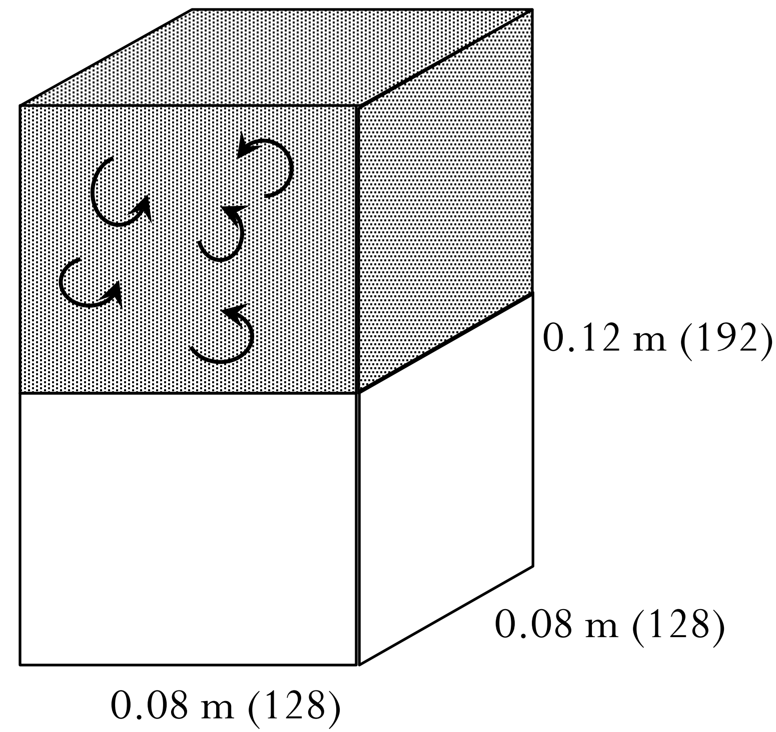
$$\nabla^2 \overline{p}^{n+1} = \nabla^2 \overline{p}_0^{n+1} - \frac{1}{\Delta t}\frac{\partial}{\partial z}\left(\overline{w}_{cd}\phi^{n+1}\right)$$

Particle Induced RT Instability

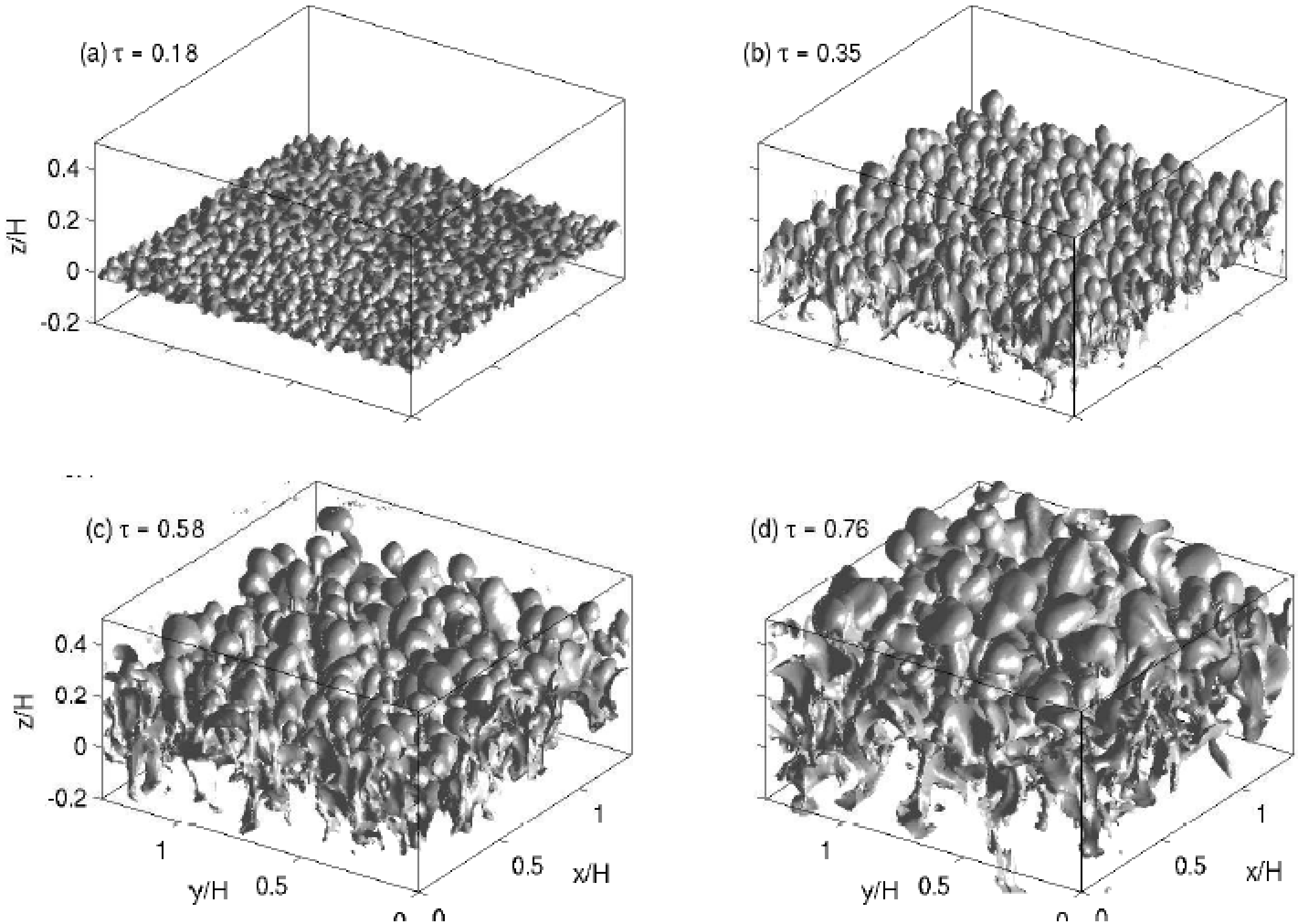
-- A numerical study of two-phase effect in suspensions

Simulation setup *(Chou et al., 2014b)*

- $\phi_0 = 0.0032, 0.0128, 0.0512$
- $d_0 = 40\ \mu m; \tau_p = 2.4 \times 10^{-4}$ sec
- Direct Numerical Simulation (DNS)
- BC: Periodic at horizontal;
  Solid wall at bottom;
  No sediment supply at top.



Growth of Initial Perturbations

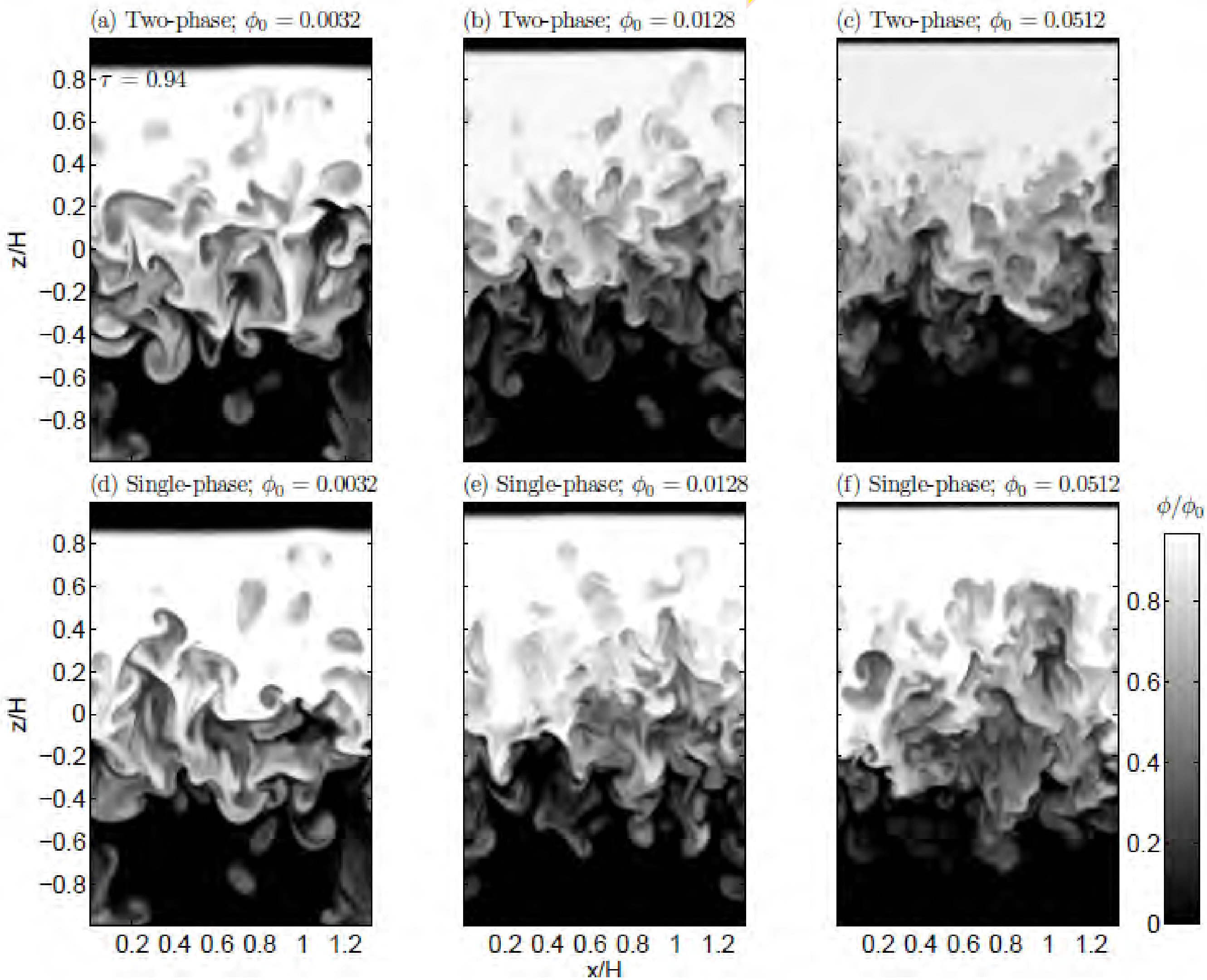


Large-Scale Mixing

Initial conc.

Present two-phase

Single phase

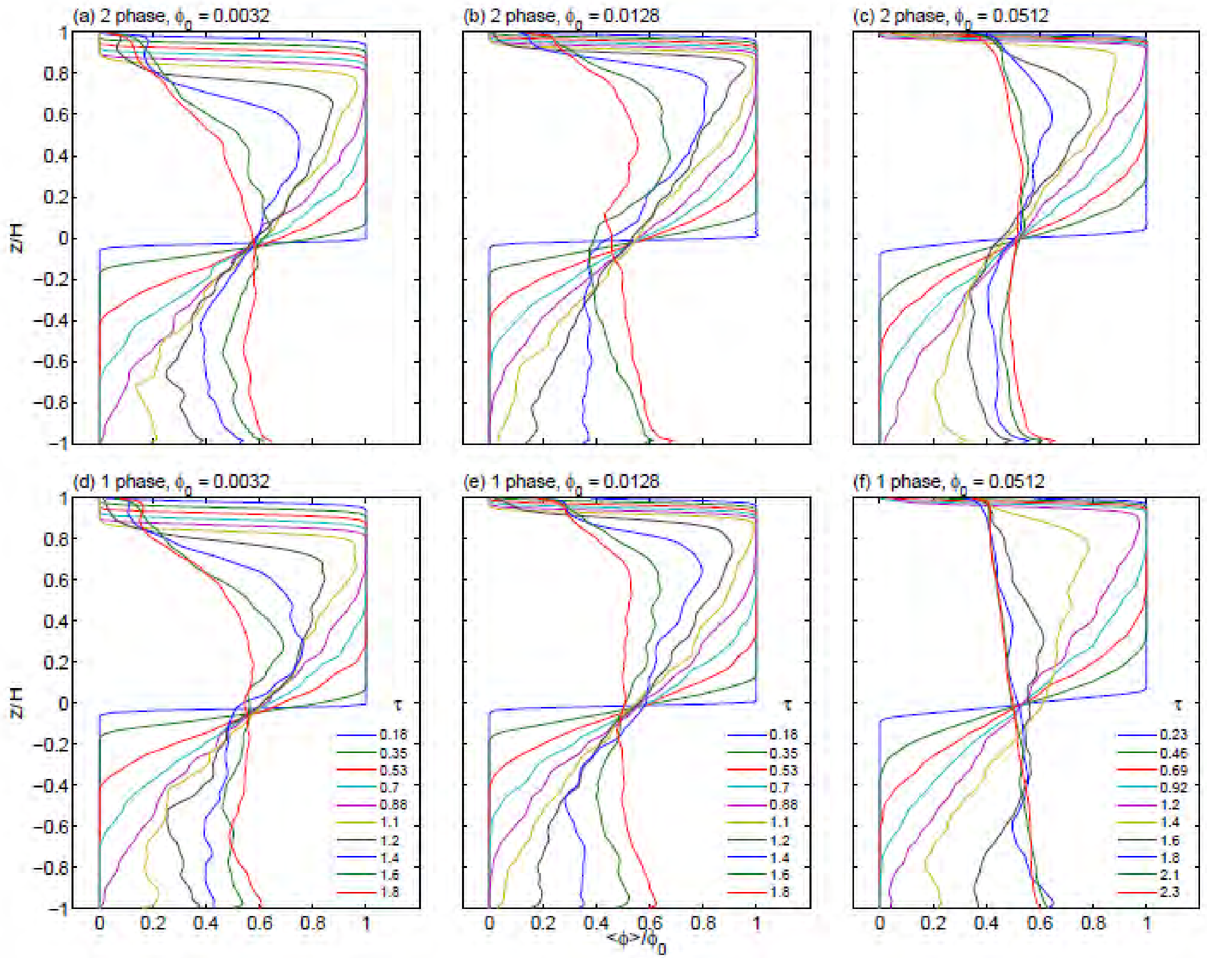


Growth of mixing zone

Initial conc.

Present two-phase

Single phase

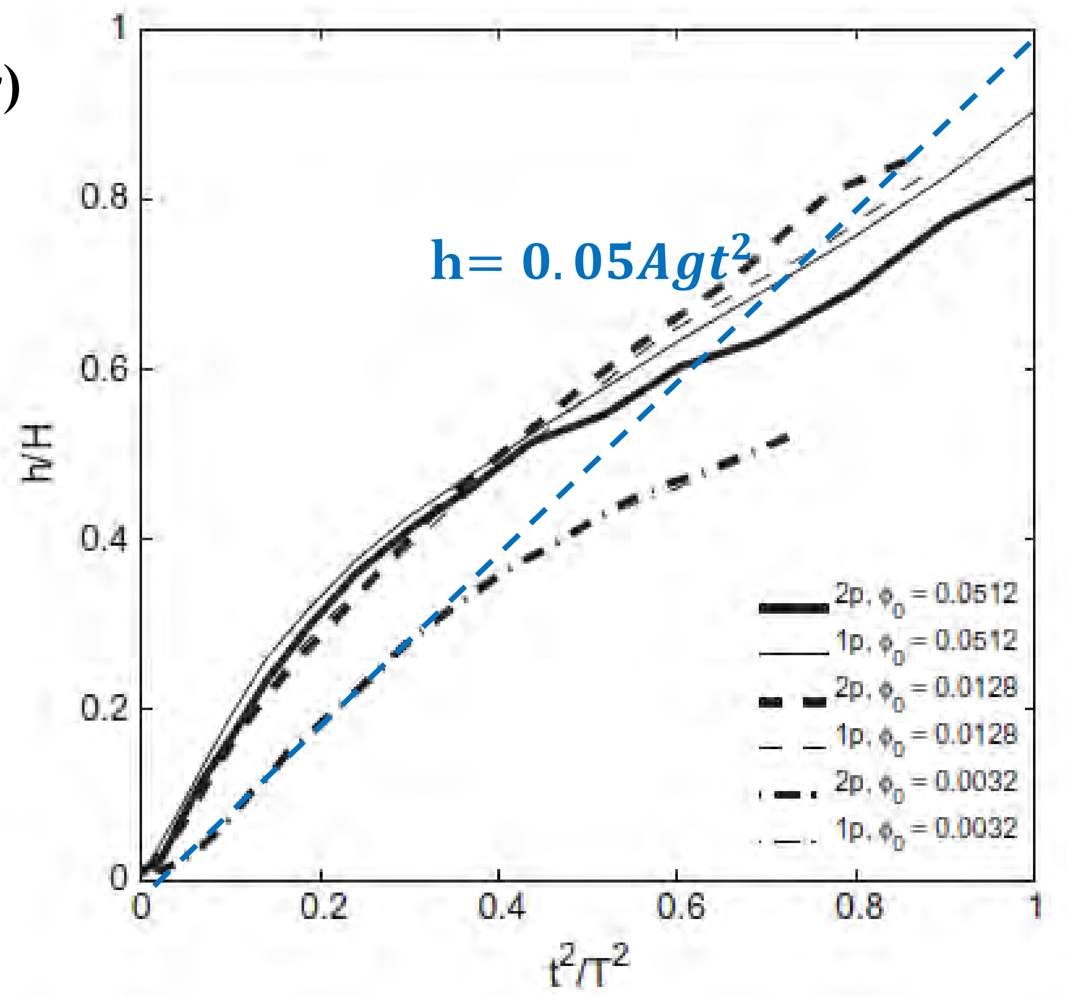


Growth of mixing zone

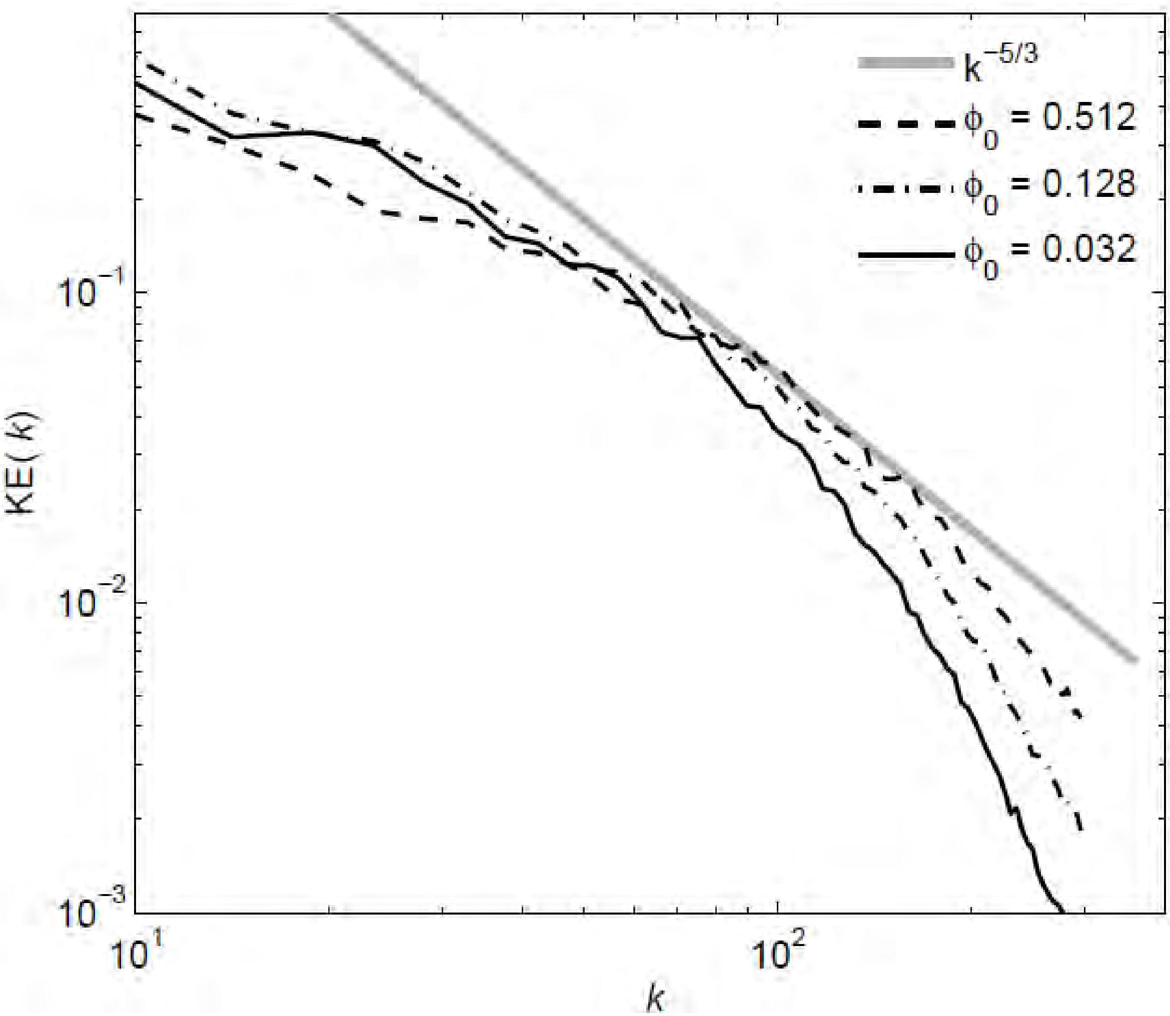
- Self-similar solution: h= $\alpha A g t^2$ (α~0.05 in the present study)

A (Atwood number)

$$= \frac{\boldsymbol{\rho_u - \rho_l}}{\boldsymbol{\rho_u + \rho_l}}$$



Energy Spectrum



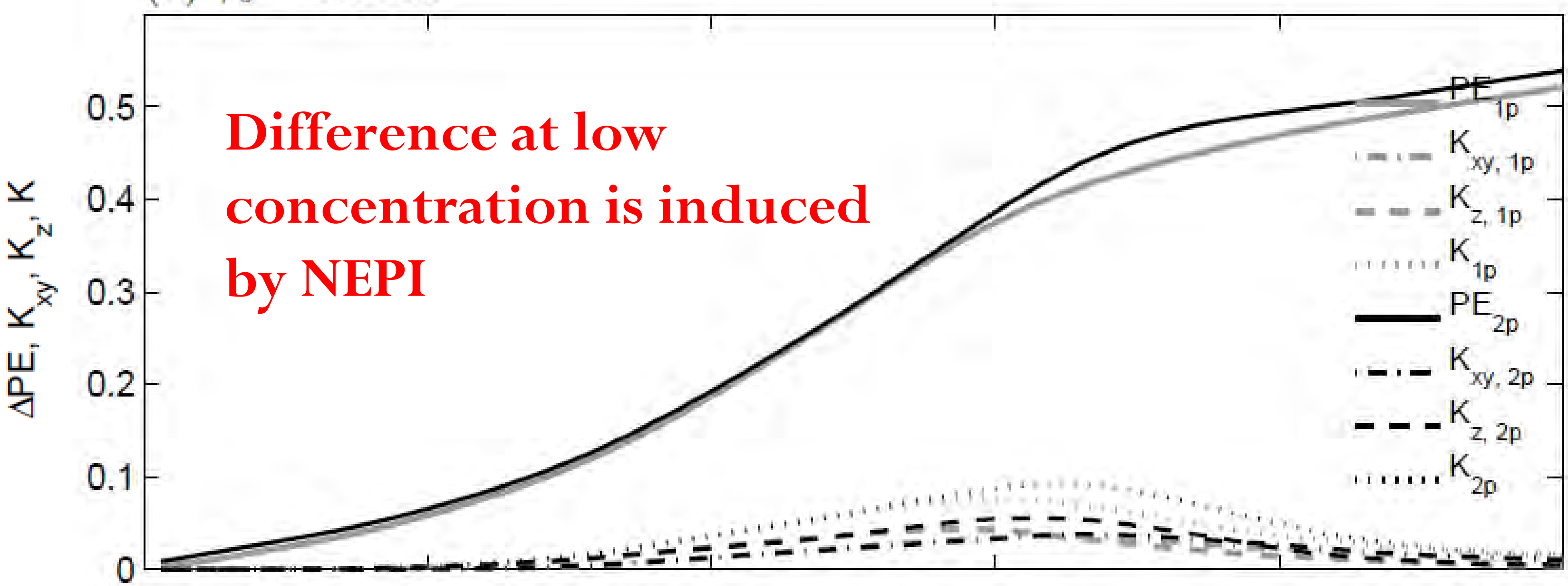
Energetic (initial conc. = 0.0032)

$$PE = \frac{1}{PE_0}\int_{-H}^{H}\int_0^W\int_0^L (\rho_s - \rho_0)\, gz dx dy dz \qquad \Delta PE = 1 - PE$$

$$K_z = \frac{1}{PE_0}\int_{-H}^{H}\int_0^W\int_0^L \rho_s\phi\overline{w}_d^2 + \rho_0(1-\phi)\,\overline{w}_c^2 dx dy dz$$

$$K_{xy} = \frac{1}{PE_0}\int_{-H}^{H}\int_0^W\int_0^L \rho_s\phi\left(\overline{u}_d^2 + \overline{v}_d^2\right) + \rho_0(1-\phi)\left(\overline{u}_c^2 + \overline{v}_c^2\right) dx dy dz$$

(a) $\phi_0 = 0.0032$

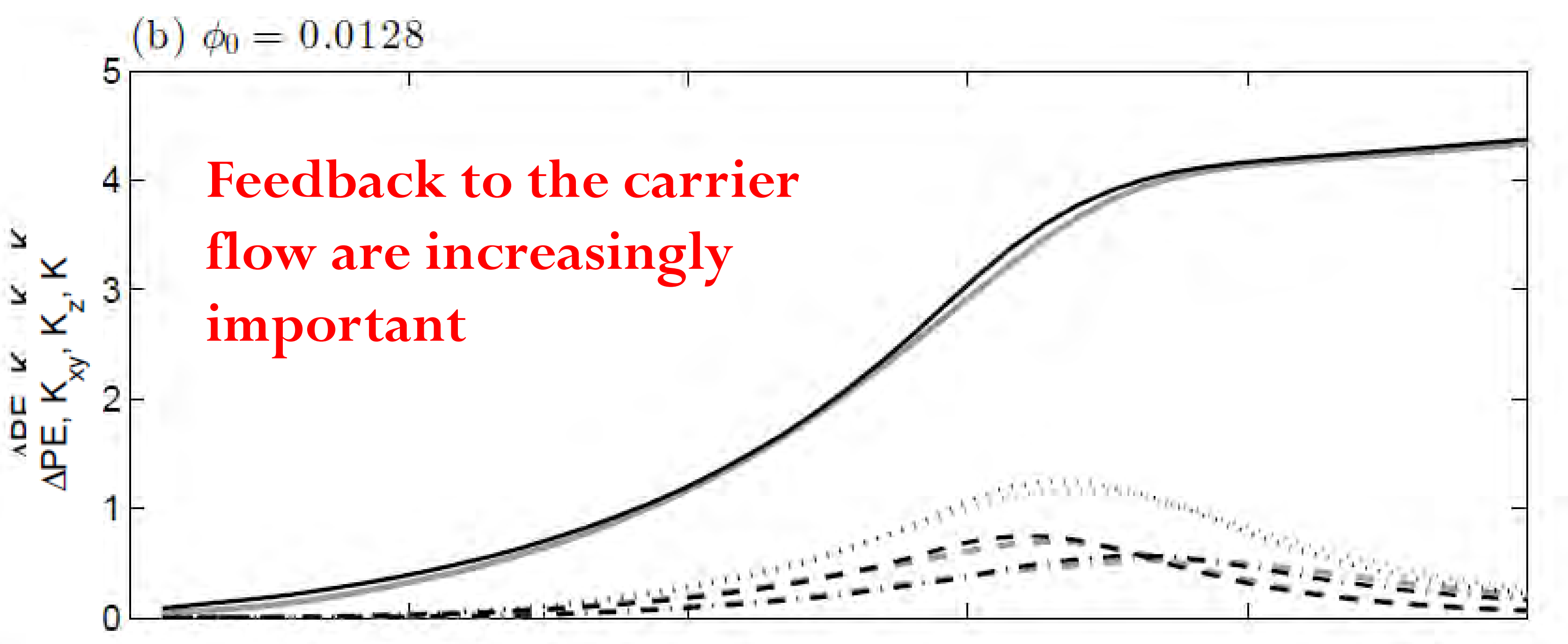


Energetic (initial conc. = 0.0128)

$$PE = \frac{1}{PE_0}\int_{-H}^{H}\int_{0}^{W}\int_{0}^{L}(\rho_s - \rho_0)\, gz dx dy dz \qquad \Delta PE = 1 - PE$$

$$K_z = \frac{1}{PE_0}\int_{-H}^{H}\int_{0}^{W}\int_{0}^{L}\rho_s \phi \overline{w}_d^2 + \rho_0 (1-\phi)\,\overline{w}_c^2 dx dy dz$$

$$K_{xy} = \frac{1}{PE_0}\int_{-H}^{H}\int_{0}^{W}\int_{0}^{L}\rho_s \phi\left(\overline{u}_d^2 + \overline{v}_d^2\right) + \rho_0 (1-\phi)\left(\overline{u}_c^2 + \overline{v}_c^2\right) dx dy dz$$

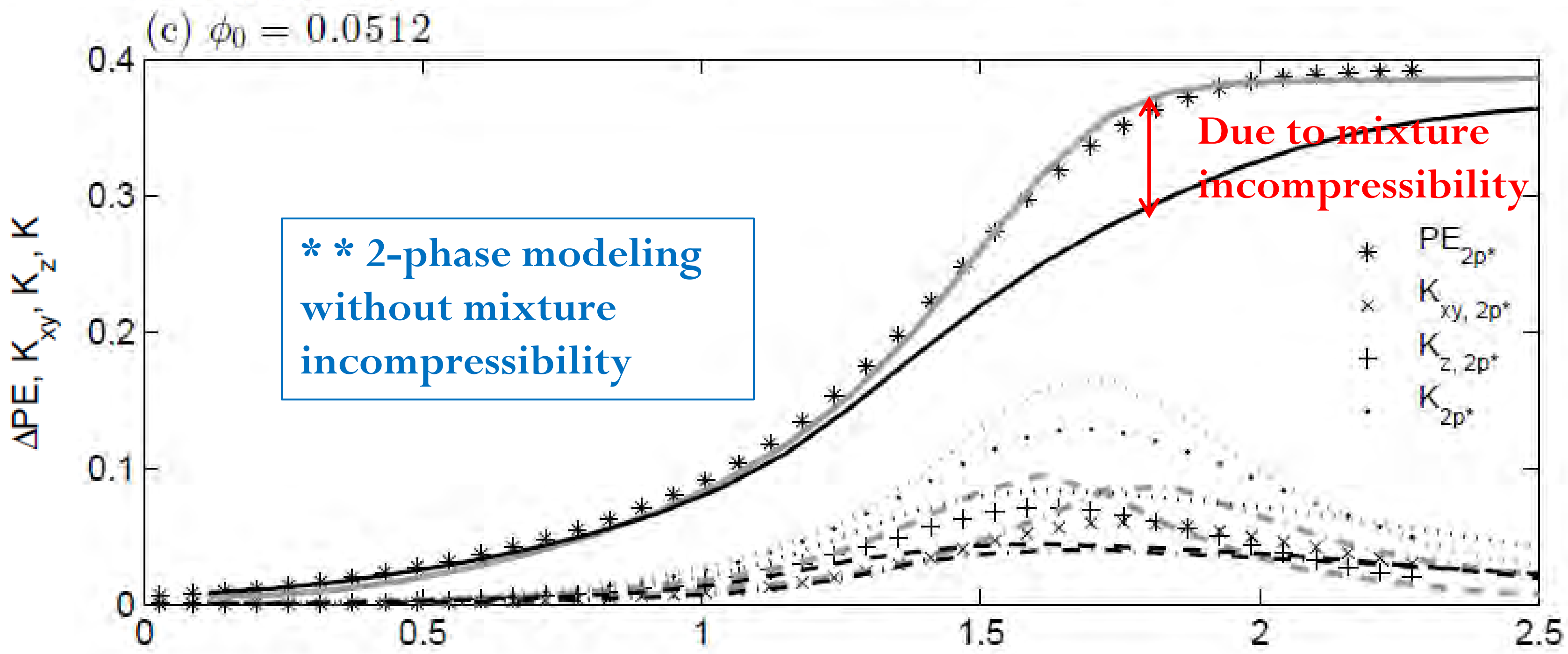


Energetic (initial conc. = 0.0512)

$$PE = \frac{1}{PE_0} \int_{-H}^{H} \int_0^W \int_0^L (\rho_s - \rho_0)\, gz dx dy dz \qquad \Delta PE = 1 - PE$$

$$K_z = \frac{1}{PE_0} \int_{-H}^{H} \int_0^W \int_0^L \rho_s \phi \overline{w}_d^2 + \rho_0 (1 - \phi)\, \overline{w}_c^2 dx dy dz$$

$$K_{xy} = \frac{1}{PE_0} \int_{-H}^{H} \int_0^W \int_0^L \rho_s \phi \left(\overline{u}_d^2 + \overline{v}_d^2\right) + \rho_0 (1 - \phi) \left(\overline{u}_c^2 + \overline{v}_c^2\right) dx dy dz$$



Summary

- We aim to investigate effects of missing mechanisms induced by two-phase interactions in the common modeling approach for sediment suspension problems.

- **The two-phase effects include:**

  -- non-equilibrium particle inertia (NEPI);

  -- NEPI in the carrier fluid;

  -- mixture incompressibility (MI)

- **A series of numerical experiments of RT reveals that**

  -- in low volume fraction, NEPI slightly enhances the energy budget

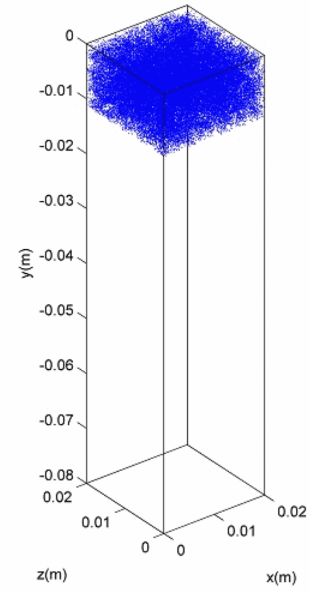
  -- as concentration increases, NEPI and MI become increasing important, which suppress energy

  -- MI is significant to suppress energy budget at high concentration, which accounts for almost ¼ of the reduction of the PE release

References:

Chou Y, Wu F, and Shih W, 2014a, "Toward numerical modeling of fine particle suspension using a two-way coupled Euler-Euler model. Part 1: Theoretical formulation and implications", *International Journal of Multiphase Flow*, in press

Chou Y, Wu F, and Shih W, 2014b, "Toward numerical modeling of fine particle suspension using a two-way coupled Euler-Euler model. Part 2: Simulation of particle-induced Rayleigh Taylor instability", *International Journal of Multiphase Flow*, in press



Thank you